

because it is flexible and well-behaved, and because it satisfies the initial conditions of zero displacement, zero velocity, and zero acceleration. The constants A , λ , and k were determined using least squares fitting. The fitted displacement function was differentiated to estimate the downward velocity as a function of time, shown as a solid curve in Figure 12–77. Velocity data points (solid circles) were also determined from the displacement data using a central difference approximation¹. The slope of the velocity curve is approximately constant between about 1.75 s and 4.0 s. To estimate the downward acceleration² during this stage, a straight line was fit to the open-circled velocity data points using linear regression (shown as a straight line in Figure 12–77). The slope of the straight line, which represents a constant acceleration, was found to be 32.2 ft/s² (with a coefficient of regression $R^2 = 0.991$), equivalent to the acceleration of gravity g . Note that this line closely matches the velocity curve between about 1.75 s and 4.0 s.

For discussion purposes, three stages were defined, as denoted in Figure 12–77:

- In Stage 1, the descent was slow and the acceleration was less than that of gravity. This stage corresponds to the initial buckling of the exterior columns in the lower stories of the north face, as seen in Figure 12–62. By 1.75 s, the north face had descended approximately 7 ft.
- In Stage 2, the north face descended at gravitational acceleration, as exterior column buckling progressed and the columns provided negligible support to the upper portion of the north face. This free fall drop continued for approximately 8 stories (105 ft), the distance traveled between times $t = 1.75$ s and $t = 4.0$ s.
- In Stage 3, the acceleration decreased somewhat as the upper portion of the north face encountered resistance from the collapsed structure and the debris pile below. Between 4.0 s and 5.4 s, the northwest corner fell an additional 130 ft.

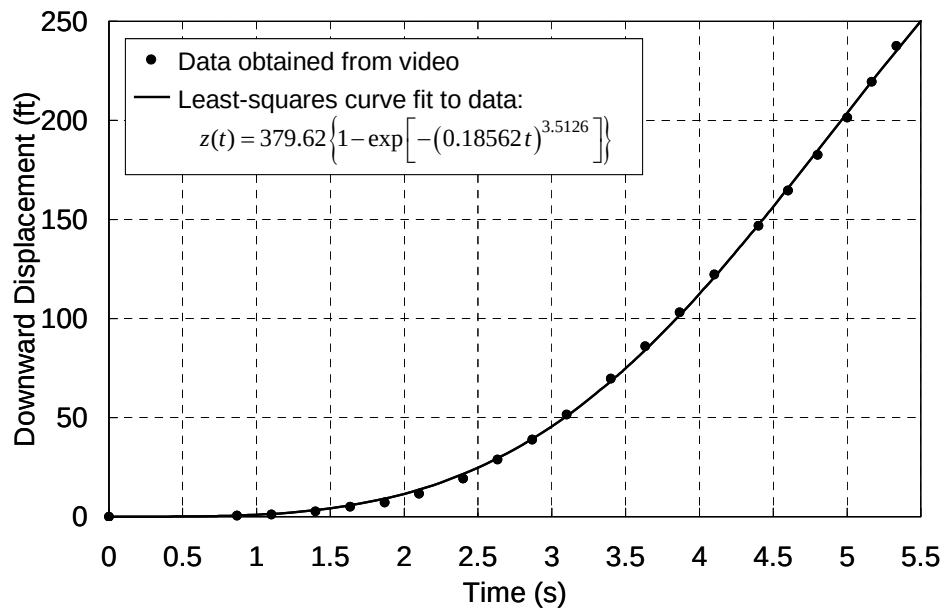


Figure 12–76
Downward
displacement of
north face roofline
as WTC 7 began to
collapse.

¹ The central difference approximation is given by $v_{i+1/2} \approx (z_i - z_{i+1})/(t_i - t_{i+1})$, where z_i and z_{i+1} denote the displacement at time t_i and t_{i+1} , respectively

² Acceleration, or rate of change of velocity, is equal to the slope of a velocity versus time curve.