

The frequency content of the data in Figure C-8 was determined using the same method described in NIST NCSTAR 1-5A, Appendix K. The time-dependent displacement of the marker point was assumed to take the form of a damped harmonic oscillator, which is described by the following equation:

$$d = [A + B(t - t_0)] + Ce^{-D(t-t_0)} \sin[2\pi(t - t_0)/E + F] \quad (1)$$

where

d = displacement, in.

t = time, s

t_0 = time at which the penthouse begins to move downward, s

A = baseline offset, in.

B = linear slope of the baseline, in./s

C = initial amplitude at time t_0 , in.

D = decay rate, (s)⁻¹

E = oscillatory period, s

F = phase angle, radian

The values to be determined were A , B , C , D , E , and F . The frequency was equal to the reciprocal of the period, E^{-1} . This mathematical model included the period, amplitude, and decay rate of the oscillations, and also included the possibility of a linear change in the zero point of the displacement with time. To accurately fit this model to the data, a good approximation was needed for each of the six variables to be determined. These approximations were then used as starting points for a nonlinear least squares fit of Eq. (1). The calculations were carried out in vertical pixel units and converted to inches. The uncertainty values given in this section do not include the conversion uncertainty.

The video showed that the east penthouse was descending between time $t = 0$ s and $t = 2.0$ s. During the time that the east penthouse was falling, the harmonic motion of the building was clearly disturbed, as can be seen in Figure C-8. The displacement of the marker point was on a plateau for the first half second after the penthouse began to fall, and the time from the minimum in the data at about -1.5 s to the maximum at about 1.7 s was about double that for oscillations before $t = 0$ s or after $t = 2.0$ s. The mathematical model assumed free harmonic motion and was not appropriate for looking at building motion when a forcing function was being applied. The time periods studied using this model were therefore from $t = -6.0$ s to $t = 0$ s and from $t = 2.0$ s to the start of collapse at $t = 6.9$ s. The motion from just after the plateau in the data at $t = 1.0$ s to collapse was also analyzed. All the results are shown in Table C-1.