

**Table C-1. Modes from nonlinear least squares analysis for WTC 7 data.**Total uncertainties are twice the standard deviation,  $\sigma$ .

| Time Range (s)                      | Period (s)      | Amplitude (in.) | Decay Rate ( $s^{-1}$ ) | Phase (radian) | Offset (in.)   | Slope (in./s)    | $2\sigma$ (in.) |
|-------------------------------------|-----------------|-----------------|-------------------------|----------------|----------------|------------------|-----------------|
| -6.0 – 0                            | $3.61 \pm 0.06$ | $1.3 \pm 0.1$   | $-0.05 \pm 0.04$        | $0.9 \pm 0.1$  | $1.0 \pm 0.1$  | $0.22 \pm 0.04$  | 0.6             |
| 2.0 – 6.9                           | $3.46 \pm 0.06$ | $12.0 \pm 1.0$  | $0.19 \pm 0.02$         | $4.7 \pm 0.1$  | $-6.6 \pm 0.5$ | $1.70 \pm 0.10$  | 0.9             |
| 2.0 – 6.9<br>(2 <sup>nd</sup> mode) | $1.65 \pm 0.02$ | $0.7 \pm 0.3$   | $0.10 \pm 0.10$         | $3.9 \pm 0.4$  | $-0.1 \pm 0.1$ | $0.03 \pm 0.03$  | 0.5             |
| 1.0 – 6.9                           | $3.75 \pm 0.04$ | $11.0 \pm 1.0$  | $0.17 \pm 0.02$         | $5.3 \pm 0.1$  | $-4.4 \pm 0.4$ | $1.20 \pm 0.10$  | 1.3             |
| 1.0 – 6.9<br>(2 <sup>nd</sup> mode) | $1.85 \pm 0.04$ | $2.0 \pm 0.4$   | $0.30 \pm 0.10$         | $5.3 \pm 0.2$  | $0.3 \pm 0.1$  | $-0.05 \pm 0.04$ | 0.7             |
| 0.5 – 6.9                           | $4.04 \pm 0.05$ | $9.0 \pm 1.0$   | $0.10 \pm 0.03$         | $5.7 \pm 0.1$  | $-2.3 \pm 0.4$ | $0.70 \pm 0.10$  | 1.8             |

For the 6 s period from the beginning of oscillatory motion in the data to when the penthouse began to fall, the harmonic motion was found to have a period of 3.61 s, an amplitude of 1.3 in., and an approximately zero decay rate. The total standard deviation from the nonlinear fit to this data (the square root of the estimated variance) was  $\sigma = 0.3$  in. The total uncertainty, with a confidence level of 95 percent, equaled twice the standard deviation in both positive and negative directions, or  $\pm 0.6$  in. Figure C-10(a) shows this mode superimposed on the original data. Subtracting the mode from the data resulted in the residual data shown in Figure C-10(b). The residual appeared random. If an oscillatory pattern had appeared in the residual, it would have suggested that a secondary vibration mode is present. In this case, the first mode found did a good job of matching the data.

The next modal analysis was performed for the time period from  $t = 2.0$  s, at the time when the penthouse was observed to completely disappear from the roof WTC 7, to  $t = 6.9$  s, when the main collapse began. The primary mode has a period of 3.46 s, an amplitude of 12 in., and a decay rate of  $0.19 s^{-1}$ . The total uncertainty was  $\pm 0.9$  in. Figure C-11(a) shows a good comparison between the mode and the data. The residual, Figure C-11(b), indicates a secondary mode with a period of 1.65 s, amplitude of 0.7 in., and zero decay rate. This is plotted in Figure C-12(a). The residual, after the combination of these two modes had been subtracted from the original data, is shown in Figure C-12(b). By adding these two modes together, the total uncertainty was reduced to  $\pm 0.5$  in., slightly better than the results for the time period before the fall of the penthouse.

To test how long the internal events within WTC 7 during the downward motion of the east penthouse drove the visible motion of the building, modal analyses were performed on wider time ranges of data, from  $t = 1.0$  s to the main collapse at  $t = 6.9$  s and from  $t = 0.5$  s to  $t = 6.9$  s. The total uncertainty for these analyses increased significantly with time range, to  $\pm 1.3$  in. for the time range of  $t = 1.0$  s to  $t = 6.9$  s and  $\pm 1.8$  in. for the time range of  $t = 0.5$  s to  $t = 6.9$  s. Figure C-13 gives visual evidence of the poorer fit for the former analysis. The discrepancies between the mode and the data were larger than in Figure C-11, and a strongly damped secondary mode was needed to fit the residual in Figure C-13(b). The modal analysis from  $t = 2.0$  s to  $t = 6.9$  s gave the best quantitative representation of the oscillatory motion of WTC 7 immediately before the final building collapse.