

The time that the roofline took to fall 18 stories or 73.8 m (242 ft) was approximately 5.4 s. The theoretical time for free fall (i.e., at gravitational acceleration) was computed from

$$t = \sqrt{\frac{2h}{g}}$$

where t = time, s; h = distance, m (ft); and g = gravitational acceleration, 9.81 m/s² (32.2 ft/s²). This time was approximately 3.9 s. Thus, the average time for the upper 18 stories to collapse, based on video evidence, was approximately 40 percent longer than the computed free fall time.

A more detailed examination of the same video led to a better understanding of the vertical motion of the building in the first several seconds of descent. NIST tracked the downward displacement of a point near the center of the roofline, fitting the data using a smooth function.³ (The time at which motion of the roofline was first perceived was taken as time zero.) The fitted displacement function was then differentiated to estimate the downward velocity as a function of time, shown as a solid curve in Figure 3-15. Velocity data points (solid circles) were also determined from the displacement data using a central difference approximation.⁴ The slope of the velocity curve is approximately constant between about 1.75 s and 4.0 s, and a good straight line fit to the points in this range (open-circles in Figure 3-15) allowed estimation of a constant downward acceleration during this time interval. This acceleration was 32.2 ft/s² (9.81 m/s²), equivalent to the acceleration of gravity g .

For discussion purposes, three stages were defined, as denoted in Figure 3-15:

- In Stage 1, the descent was slow and the acceleration was less than that of gravity. This stage corresponds to the initial buckling of the exterior columns in the lower stories of the north face. By 1.75 s, the north face had descended approximately 2.2 m (7 ft).
- In Stage 2, the north face descended at gravitational acceleration, as the buckled columns provided negligible support to the upper portion of the north face. This free fall drop continued for approximately 8 stories or 32.0 m (105 ft), the distance traveled between times $t = 1.75$ s and $t = 4.0$ s.
- In Stage 3, the acceleration decreased somewhat as the upper portion of the north face encountered increased resistance from the collapsed structure and the debris pile below. Between 4.0 s and 5.4 s, the north face corner fell an additional 39.6 m (130 ft).

As noted above, the collapse time was approximately 40 percent longer than that of free fall for the first 18 stories of descent. The detailed analysis shows that this increase in time is due primarily to Stage 1. The three stages of collapse progression described above are consistent with the results of the global collapse analyses discussed in Chapter 12 of NIST NCSTAR 1-9.

³ A function of the form $z(t) = A\{1 - \exp[-(t/\lambda)^k]\}$ was assumed, which satisfies the initial conditions of zero displacement, zero velocity, and zero acceleration. The constants A , λ , and k were determined using least squares fitting.

⁴ The central difference approximation is given by $v_{i+1/2} \approx (z_i - z_{i+1})/(t_i - t_{i+1})$, where z_i and z_{i+1} denote the displacement at time t_i and t_{i+1} , respectively.